

Quantum Field Theory Vector Spaces

Operators

Physical quantities like energy, electric charge, momentum, spin, weak isospin charge, QCD charge

H, Q, \mathbf{P} , etc. on states
 $\Sigma_3, \sigma_i/2$, etc. on fields

Fields

Solutions to field equations that create and destroy particles

ϕ, ψ, A^μ

States

Physical particles, one or more (though vacuum state has none)

$|\text{particles}\rangle$
 like $|e^+\rangle$ or $|e^+ e^-\rangle$

Operators \longleftarrow A vector space and operators \longrightarrow Vectors

Operators \longleftarrow Another vector space and operators \longrightarrow Vectors

Operators \longleftarrow Yet another vector space and operators \longrightarrow Vectors

Example $\Sigma_3 \psi_{\mathbf{p}=0}^{spinup} = \frac{1}{2} \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix} \left[c_r(\mathbf{p}) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ipx} + d_r^\dagger(\mathbf{p}) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ipx} \right] = \frac{1}{2} c_r(\mathbf{p}) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ipx} + d_r^\dagger(\mathbf{p}) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ipx} = \frac{1}{2} \psi_{\mathbf{p}=0}^{spinup}.$

Example $\text{QFT } \Sigma_3 \left| e_{up}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle = \left(\int_V \psi^\dagger \Sigma_3 \psi d^3x \right) \left| e_{up}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle$

$$= \sum_{r, \mathbf{p}} \frac{m}{E_{\mathbf{p}}} \left(u_r^\dagger(\mathbf{p}) \Sigma_3 u_r(\mathbf{p}) N_r(\mathbf{p}) + v_r^\dagger(\mathbf{p}) \Sigma_3 v_r(\mathbf{p}) \bar{N}_r(\mathbf{p}) \right) \left| e_{up}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle = \frac{1}{2} \left| e_{up}^-(r=1) \right|_{\mathbf{p}_1=0} \rangle$$